final wealth after one trading day

$1000(1+R_t)$

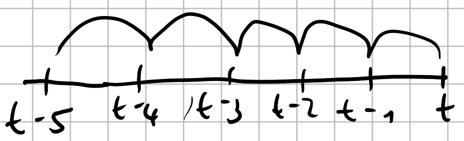
$P(\underbrace{1000(1+R_t)\le 990})$ $\qquad r_t \overset{iid}{\sim} N(\overbrace{0.001}^{\mu}, \overbrace{0.015^2}^{\sigma^2})$

$$1000(1+R_t)\le 990 \Rightarrow 1+R_t \le 0.99 \Rightarrow \log(1+R_t) \le \log(0.99)$$

$$P(r_t \le \log(0.99)) = P\left(\frac{r_t-0.001}{0.015} \le \frac{\log(0.99)-0.001}{0.015}\right)$$

$\sim N(0,1)$ (refer to standard normal tables)

≈ 0.23



final wealth = $1000(1+R_t(5))$

$P(1000(1+R_t(5))\le 990)$

$$1+R_t(5)\le 0.99 \Rightarrow \underbrace{\log(1+R_t(5))}_{r_t(5)} \le \log(0.99)$$

$$r_t(5) = r_t + r_{t-1}+r_{t-2}+r_{t-3}+r_{t-4}$$

$P(r_t+\dots+r_{t-4} \le \log(0.99))$ $\qquad r_t+r_{t-1}+\dots+r_{t-4} \sim N(\overset{k\mu}{5\cdot 0.001}, \underset{k\sigma^2}{5\cdot 0.015^2})$

Practice exercises 2, 3, $\underbrace{4, 5, 6}_{\text{no RW model needed here}}$, 7a, 7b

How to do Exercise 1 completely using a computer?

Rely on section 2.4.3

$$P_t = P_0\exp(r_t+r_{t-1}+\dots+r_1)$$

$P_0, \quad P_1 = P_0\exp(r_1), \quad P_2 = P_0\exp(r_1+r_2), \dots$

$r_t, \dots, r_{t-k+1} \overset{iid}{\sim} N(\mu, \sigma^2)$

$$r_t + r_{t-1} + \dots + r_{t-k+1} \sim N(k\mu, k\sigma^2)$$

$0.2^2 = 253\sigma^2$

$0.2/\sqrt{253} = \sigma$

$$\underbrace{E(r_t+\dots+r_{t-k+1})}_{0.05} = k\mu$$

$$0.05 = 253\mu \Rightarrow \mu = 0.05/253$$

Exercise 7

r_t = log return

$r_1, r_2, \ldots \overset{iid}{\sim} N(0.06, 0.47)$

$r_t(k) = r_t + r_{t-1} + \ldots + r_{t-k+1}$

(c) $Cov(r_2(1), r_2(2))$

$r_2(1) \Downarrow r_2$, $r_2(2) \Downarrow r_2 + r_1$

X, Y indpt $\Rightarrow$ $Cov(X,Y) = 0$

$= Cov(r_2, r_2 + r_1) = Cov(r_2, r_2) + Cov(r_2, r_1) = \underline{Var(r_2)} + 0$

$Cov(X,X) = Var(X)$

$Var(r_2) = 0.47$

X, Y, Z r.v. $\quad Cov(X, Y+Z) = Cov(X,Y) + Cov(X,Z)$

(d) $r_t(3) \mid r_{t-2} = 0.6$

$r_{t-2} + r_{t-1} + r_t \mid \underline{r_{t-2} = 0.6} \quad \sim N(\quad , \quad)$

$Var(r_{t-2} + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$\mathbb{E}(r_{t-2} + r_{t-1} + r_t \mid \underline{r_{t-2} = 0.6})$

$= \mathbb{E}(\underline{0.6} + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= 0.6 + \underline{\mathbb{E}(r_{t-1} \mid r_{t-2} = 0.6)}$

$+ \underline{\mathbb{E}(r_t \mid r_{t-2} = 0.6)}$

r's are indpt

$\overset{\downarrow}{=} 0.6 + \mathbb{E}(r_{t-1}) + \mathbb{E}(r_t)$

$= 0.6 + 2(0.06) = \_\_\_$

$Var(r_{t-2} + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= Var(0.6 + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= Var(r_{t-1} + r_t \mid \underline{r_{t-2} = 0.6})$

$= Var(\underbrace{r_{t-1} + r_t}_{\text{indpt}}) = Var(r_{t-1}) + Var(r_t) = 2(0.47)$