#8

(a) $\mathbb{P}\left(X_2 > 1.3 X_0\right)$

$$= \mathbb{P}\left(\frac{X_2}{X_0} > 1.3\right) = \mathbb{P}\left(\exp(r_1+r_2) > 1.3\right)$$

$$= \mathbb{P}\left(r_1+r_2 > \log(1.3)\right) = \_\_\_\_$$

(b) density of $X_1 = \overset{\text{constant}}{X_0} \exp(\underbrace{r_1}_{\text{dist.}})$ $\qquad r_1 \sim N(\mu, \sigma^2)$

a function of r_1

(A.4) $\overbrace{f_Y(y)}^{\text{density of transformed r.v.}} = f_X(h(y)) \, \boxed{|h'(y)|}$

transformed r.v. — old r.v. — density of old r.v.

$h(\cdot) \to$ inverse of the transformation — exists.

$Y = g(X)$

$\to X = h(Y)$

$\boxed{X_1} = X_0 \exp(r_1)$

transformed r.v. — old r.v.

To find inverse:

$$\frac{X_1}{X_0} = \exp(r_1) \Rightarrow r_1 = \log\left(\frac{X_1}{X_0}\right)$$

$$r_1 = \underbrace{\log X_1 - \log X_0}_{h(X_1)}$$

Derivative:

$$\frac{dr_1}{dX_1} = \frac{1}{X_1}$$

Density of old r.v. $\quad r_1 \sim N(\mu, \sigma^2)$

$$f_{r_1}(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{s-\mu}{\sigma}\right)^2\right)$$

X_1 r.v.

x_1 a value of X_1

$$f_{X_1}(x_1) = f_{r_1}(h(x_1)) \, |h'(x_1)|$$

since $X_1 = X_0 \exp(r_1)$, $X_0 > 0$, $\exp(r_1) > 0$

$$= f_{r_1}(\log x_1 - \log X_0) \left|\frac{1}{x_1}\right|$$

$$= \frac{1}{x_1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\log x_1 - \log X_0 - \mu}{\sigma}\right)^2\right)$$

$$= \underbrace{\frac{1}{x_1\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\log x_1 - (\log X_0 + \mu)}{\sigma}\right)^2\right)}_{\text{density of } X_1 \sim \text{lognormal}(\log X_0 + \mu,\ \sigma^2)}$$

named distributions / densities

From wiki: $\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)$

pattern matching

density of a lognormal(μ, σ^2)

$\Rightarrow \quad X_1 = X_0 \exp(r_1)$ where $r_1 \sim N(\mu, \sigma^2)$

density of X_1 is going to be lognormal$(\log X_0 + \mu, \sigma^2)$

There is an easier way to do this!

How is lognormal related to normal?

$\underline{Z \sim N(\mu, \sigma^2)} \Rightarrow Y = \exp(Z) \sim$ lognormal(μ, σ^2)

$\log Y = Z$

(*) $X_1 = X_0 \exp(r_1)$

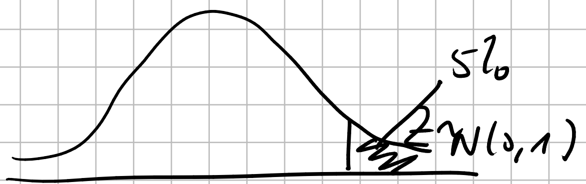
$\underline{\log X_1} = \underbrace{\log X_0}_{\text{constant}} + \underbrace{r_1}_{\sim N(\mu, \sigma^2)}$

$\sim \underline{N}(\underline{\log X_0 + \mu}, \underline{\sigma^2})$

Therefore, $X_1 = X_0 \exp(r_1)$ is lognormal $(\log X_0 + \mu, \sigma^2)$.

dist of $X_k = X_0 \exp(r_1 + \dots + r_k)$

quantiles



$\underline{P(Z \le ?)} = \underline{0.95}$

? = 0.95 quantile (95th percentile)

$P(X_k \le x) = 0.9$ $\qquad$ $x \Rightarrow$ 0.9 quantile

$P(\underbrace{X_0}_{\text{constant}} \exp(r_1 + \dots + r_k) \le x) = 0.9$

$P(r_1 + \dots + r_k \le \boxed{\log\left(\frac{x}{X_0}\right)}) = 0.9$

Chapter 4 → Exploratory* data analysis (univariate)
- 4.1 – 4.5

How is this different from what you saw in ECONMETR?
↳ regressions
- descriptive (summary involves more than 1 variable)
- predictive
- causal

} conditional dist. joint dist.

statistical tools which can be used to highlight features of the data

target: understand / explore features of marginal* distribution of some r.v.

time series plots, histograms, density estimates, probability plots (QQ-plots), boxplots

$\frac{1}{n}\sum X_i \xrightarrow{P} E(X_i)$

For all n, m: $(Y_1, \_\_, Y_n)$ has the same dist as $(Y_{1+m}, \ldots, Y_{n+m})$

set $m=1$, $n=1$

Y_1 has the same dist as Y_2

set $m=2$ $n=1$

Y_1 has the same dist as Y_3

Set $m=1$, $n=2$

(Y_1, Y_2) has same dist as (Y_2, Y_3)