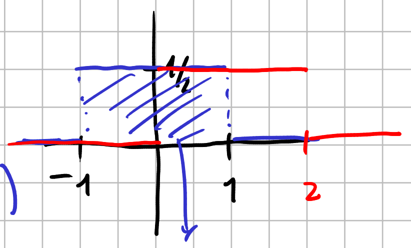
t_ν dist has a zero ~~mean~~ population mean (if it exists), but the data (PSE: return dist) indicate that returns have a nonzero sample mean.

p. 95 Suggestion as to how to allow for nonzero population means.

$X \sim t_\nu$ transform into $Y = \mu + \lambda X$ (altering letters relative to what you see in the book)

μ: location shift, λ: scaling factor

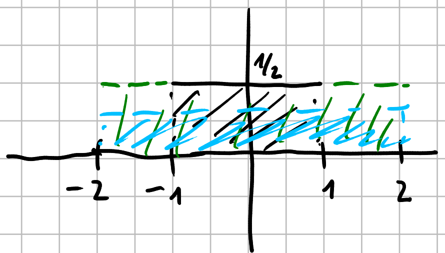
example
uniform dist.
$X \sim U(-1,1)$
$\mathbb{E}(X) = 0$



transform Y

$\mathbb{E}(Y) = 1$

area has to be equal to 1.



$Y = \underset{a}{\mu} + \underset{b}{\lambda} X$

Result A.1

$$f_Y(y) = \frac{1}{|\lambda|} f_X\left(\frac{y-\mu}{\lambda}\right)$$

f_X: Density of t_ν dist.

$X \sim t_\nu$ apply $Y = \mu + \lambda X \sim t_\nu(\mu, \lambda^2)$

loc: μ, scale: λ^2, ν: df / heaviness of tail

* Software implementations vary with respect to how distributions are formulated and coded.

e.g. dnorm(__ , mean = __ , sd = __) $N(\mu, \sigma)$

In R (fGarch package), dstd( ) → density of standardized t-dist.

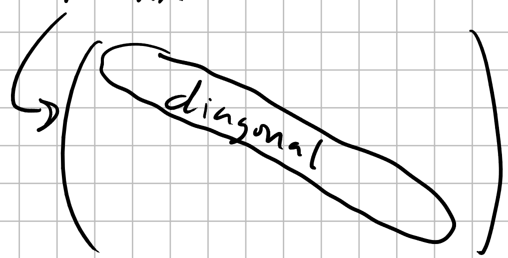
5.10 θ one dimensional 1×1

$$\mathcal{I}(\theta) = -\mathbb{E}\left(\frac{d^2}{d\theta^2} \log L(\theta)\right)$$

$$se(\hat\theta) = \frac{1}{\sqrt{\mathcal{I}(\hat\theta)}} = \sqrt{\mathcal{I}^{-1}(\hat\theta)}$$

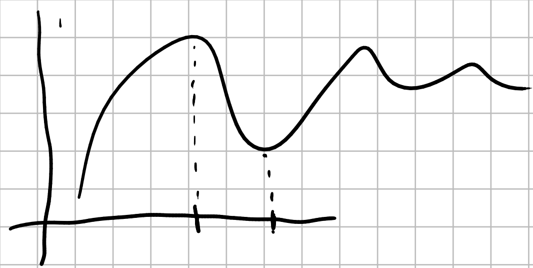
→ θ multidimensional

Fisher info matrix



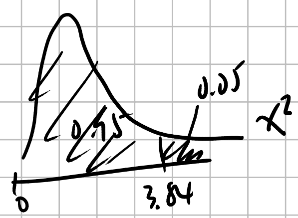
SE for each estimated qty ⟵ sqrt(reciprocal of each diag entry)

at the MLE, log likelihood function should look quadratic
(around)



introduce asymmetry (lack of symmetry) for t-dist

dsstd()
↑ skewed.



$H_0: \xi = 1$

$$\frac{\hat{\xi} - 1}{se(\hat{\xi})}$$

likelihood ratio test

$H_0: \xi = 1$ $\quad df = 1$ (1 restriction)

- estimate by MLE without imposing $\xi = 1$
- get log likelihood value
- estimate by MLE with $\xi = 1$ imposed
- get log likelihood value