Can we apply OLS in cases where we have time series data?

↳ lose independence

t	Y_t	X_t
1		–
2		–
⋮		
T		

Example $Y_t = \beta_0^* + \beta_1^* Y_{t-1} + u_t$ $\quad u_t \sim N(0,1)$, $Y_0 \sim N(0,1)$

lm($Y_t \sim Y_{t-1}$)

ECOFMET → heavy-tailed dist $\quad u_t \sim t_3$

Example $Y_t = \beta_0^* + \beta_1^* Y_{t-1} + \beta_2^* Y_{t-2} + u_t$ $\quad$ AR(2)

lm($Y_t \sim Y_{t-1}$)

a common situation in practice: because we do not know the model that generated the data

hard to obtain knowledge about the dynamic specification

---

unit root case

$$Y_t = Y_{t-1} + u_t$$
$$Y_1 = Y_0 + u_1$$
$$Y_2 = Y_1 + u_2 = Y_0 + u_1 + u_2$$
$$Y_3 = Y_2 + u_3 = Y_0 + u_1 + u_2 + u_3$$
$$\vdots$$
$$Y_t = Y_0 + \boxed{\underbrace{u_0 + u_1 + u_2 + \dots + u_t}_{\text{accumulation of errors.}}}$$

stochastic trend

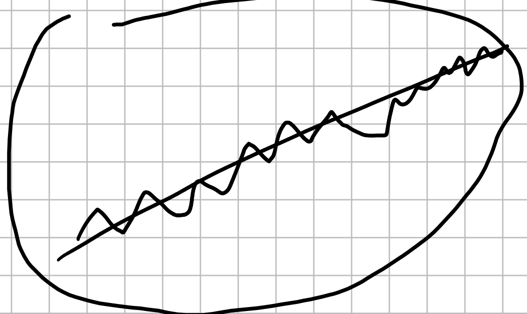
"remember" past errors completely

$0 < \beta_1^* < 1$

$$Y_t = \beta_1^* Y_{t-1} + u_t$$
$$Y_1 = \boxed{\beta_1^* Y_0} + u_1$$
$$Y_2 = \beta_1^* Y_1 + u_2$$
$$= \beta_1^*(\beta_1^* Y_0 + u_1) + u_2$$
$$= (\beta_1^*)^2 Y_0 + \beta_1^* u_1 + u_2$$
$$Y_3 = (\beta_1^*)^3 Y_0 + (\beta_1^*)^2 u_1 + \beta_1^* u_2 + u_3$$
$$\vdots$$
$$Y_t = (\beta_1^*)^t Y_0 + (\beta_1^*)^{t-1} u_1 + (\beta_1^*)^{t-2} u_2 + \dots + \beta_1^* u_{t-1} + u_t$$

decay ← "forget" past errors ⇐

before late 70s → macroeconomists believed that macro time series have deterministic trend



First differences $= Y_t - Y_{t-1} = \Delta Y_t$

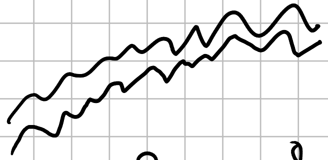
$Y_t = 0.1 + Y_{t-1} + \varepsilon_t \Rightarrow \underbrace{Y_t - Y_{t-1}}_{\Delta Y_t} = 0.1 + \varepsilon_t$

$X_t = 0.1 + X_{t-1} + \eta_t$

$\Rightarrow \underbrace{X_t - X_{t-1}}_{\Delta X_t} = 0.1 + \eta_t$

lm($\Delta Y \sim \Delta X$)

Trending series may have a common stochastic trend.



Example $Y_t = \overset{0}{\beta_0^*} + \overset{0.8}{\beta_1^*} Y_{t-1} + \overset{-0.2}{\cancel{\beta_2^*}} Y_{t-2} + u_t$ ✓ AR(2)

lm($Y_t \sim Y_{t-1}$)

coef Y_{t-1}
se

lm($Y_t \sim Y_{t-1} + Y_{t-2}$)

coef Y_{t-1}
se

lm($Y_t \sim Y_{t-1}$) $\Rightarrow$ I will learn $\frac{\beta_1^*}{1-\beta_2^*}$ in the limit.

. .

* Are returns predictable?
↳ excess returns.

risk free rate

$$r_t = \beta_0 + \beta_1 r_{t-1} + u_t$$

$$\mathrm{lm}(r_t \sim r_{t-1})$$

$$\hat{\beta}_1 = 0.19$$

$$H_0: \beta_1 = 0$$

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + u_t$$

$$H_0: \beta_1 = 0, \beta_2 = 0$$