IID $\rightarrow$ dependence + stationarity

Stock & Watson (SW)

Ruppert & Matteson (SDAFE)

↳ one concept of stationarity

↳ strict stationarity $\rightarrow$ distribution of blocks of r.v.'s unchanging over time

weak stationarity $\rightarrow$ mean, variance, and covariance unchanging over time

moments exist + strict stationary $\Rightarrow$ weak stationarity

$$\underbrace{\mathrm{Cov}(Y_t, Y_s)} = \gamma(|t-s|) \qquad \gamma(h)$$

function of $|t-s|$

↳ covariances are function of $|t-s|$

$t-s$ = gap between two time periods.

$\bar{Y}$ Sample average

How are the properties of sample average going to be affected by the change from IID to dependence + stationarity?

$$\bar{Z} = \frac{Z_1 + Z_2 + \dots + Z_n}{n}$$

n = # of obs / r.v.'s

$Z_1, \dots, Z_n$ dependence ✓ stationarity finite moments

$$= \frac{1}{n}\sum_{t=1}^{n} Z_t$$

$$\mathbb{E}(\bar{Z}) = \frac{1}{n}\left(\mathbb{E}(Z_1) + \mathbb{E}(Z_2) + \dots + \mathbb{E}(Z_n)\right)$$

By stationarity

$\mathbb{E}(Z_1) = \mathbb{E}(Z_2) = \dots = \mathbb{E}(Z_n) = \mathbb{E}(Z_t) = \mu$

doesn't depend on t

$$= \frac{1}{n}\big(\underbrace{\mathbb{E}(Z_t) + \mathbb{E}(Z_t) + \dots + \mathbb{E}(Z_t)}_{n \text{ copies.}}\big)$$

$$= \mathbb{E}(Z_t) = \mu$$

$Z_1, \dots, Z_n$ IID (μ, σ^2)

ECON STA

$\boxed{\mathbb{E}(\bar{Z}) = \mu}$

$\mathrm{Var}(\bar{Z}) = \sigma^2/n$

$\bar{Z} \xrightarrow{P} \mu$

converges in prob.

$$\mathrm{Var}(\bar{Z}) = \mathrm{Var}\left(\frac{1}{n}(Z_1 + \dots + Z_n)\right)$$

constant

$$= \frac{1}{n^2}\mathrm{Var}(Z_1 + \dots + Z_n)$$

variance of a sum of r.v.'s?

$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y)$

dependence $\stackrel{?}{=} \frac{1}{n^2}\Big[Var(Z_1) + Var(Z_2) + \dots + Var(Z_n)$
$+ 2Cov(Z_1, Z_2)$
$+ 2Cov(Z_1, Z_3)$
$+ 2Cov(Z_1, Z_4)$
$\vdots \quad + \dots \Big]$

$Var(X+Y+Z) = Var(X) + Var(Y) + Var(Z) + 2Cov(X,Y) + 2Cov(X,Z) + 2Cov(Y,Z)$

organize as matrix

$$Var\left(\frac{Z_1+Z_2}{2}\right) = \frac{1}{4}\Big[\underbrace{Var\, Z_1 + Var(Z_2) + \boxed{2Cov(Z_1, Z_2)}}\Big]$$

$$\begin{pmatrix} Var(Z_1) & \boxed{Cov(Z_1,Z_2)} \\ \boxed{Cov(Z_2,Z_1)} & Var(Z_2) \end{pmatrix}$$

$$\begin{bmatrix} Var(Z_1) & Cov(Z_1,Z_2) & Cov(Z_1,Z_3) & Cov(Z_1,Z_4) & \dots & Cov(Z_1,Z_n) \\ Cov(Z_2,Z_1) & Var(Z_2) & Cov(Z_2,Z_3) & Cov(Z_2,Z_4) & \dots & Cov(Z_2,Z_n) \\ Cov(Z_3,Z_1) & Cov(Z_3,Z_2) & \ddots & & & \\ \vdots & & & & & \\ Cov(Z_n,Z_1) & Cov(Z_n,Z_2) & \dots & & & Var(Z_n) \end{bmatrix}$$

under stationarity:

not equal

$Cov(Z_1, Z_2) = Cov(Z_2, Z_3) = \dots$ one period apart

$Cov(Z_1, Z_3) = Cov(Z_2, Z_4) = \dots$ two periods apart

ELINSTA

$Var(\bar{Z}) = \sigma^2/n \Rightarrow$ Standard error of $\bar{Z} = \sigma/\sqrt{n}$

ELWFMET

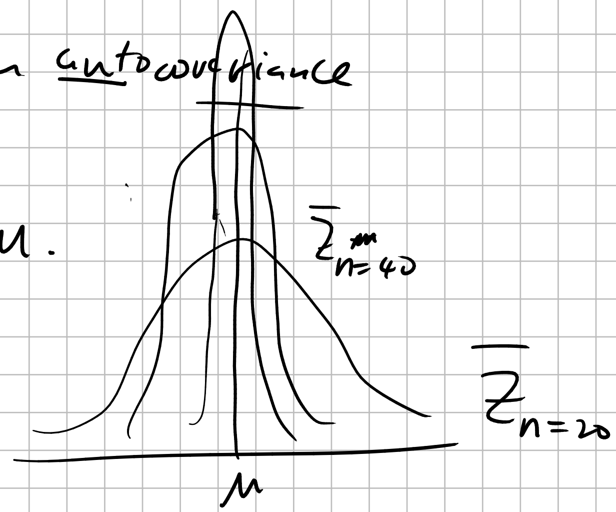
$Var(\bar{Z}) = \frac{\sigma^2}{n} + \bigcirc$

$Cov(Z_t, Z_{t-j})$ j^{th}-order autocovariance

ECONSTA Under IID, $\bar{Z} \xrightarrow{p} \mu$.

$Var(\bar{Z}) = \frac{\sigma^2}{n} \to 0$ As $n \to \infty$

As $n \to +\infty$



$1 + 2 + 3 + 4 + \cdots$ = not finite sum

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ = not finite sum

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$Y_t = 1 \cdot Y_{t-1} + u_t$ random walk

$Y_t = \beta_1^* Y_{t-1} + u_t$ $\quad 0 < \beta_1^* < 1$

parallels to ECOMETR

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$$

$$E(\varepsilon_t | X_t) = 0$$

IID, $\cancel{Var(\varepsilon_t | X_t) = \sigma^2}$ cond. homoscedasticity

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dependence, stationarity $\Rightarrow$ effects on sample average.

autocovariances may not necessarily be equal to zero

formulas learned in ECONSTA / ECOMETR may not apply.

What kind of time series models can we entertain?
and how can they be useful?

SW: fitting autoregressions in order to produce forecasts, test hyp...

SDAFE: figuring out what type of time series model fits the data, forecasting is more of an afterthought

given time series → AR? MA? ARMA?

$1, 2, \ldots, T$ → $T+1$

what you observe — what you do not observe yet

parallels to ECONOMETR

	ECOFMET		ECONMETR	
What you see	$Y_1, \ldots, Y_T$	What you see	$X_1, X_2, \ldots, X_k$	regressors / features
What you want to predict	Y_{T+1}	what you want to predict	Y	
best prediction in terms of the smallest mean squared error	$E(Y_{T+1} \mid Y_1, \ldots, Y_T)$		$E(Y \mid X_1, \ldots, X_k)$	function of features / regressors.

function of past data

general functions (they are completely unspecified)

SW Section 15.3

$$E(Y_{T+1} \mid Y_T, \ldots, Y_1) = \beta_0 + \beta_1 Y_T$$

$$* \quad \underline{E(Y_t \mid Y_{t-1}, Y_{t-2}, \ldots)} = \underline{\beta_0 + \beta_1 Y_{t-1}} \quad *$$

similar to ECONMETR $\underline{E(Y \mid X_1, \ldots, X_k)} = \beta_0 + \underline{\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k}$

cond. exp = best prediction!

$$\mathbb{E}(u_t \mid Y_{t-1}, Y_{t-2}, \dots) = 0$$

past things you observe as of time t

$$\boxed{Y_t} = \boxed{\beta_0 + \beta_1 Y_{t-1}} + \boxed{u_t}$$

$$\mathbb{E}(Y_t \mid Y_{t-1}, \dots) = \beta_0 + \beta_1 Y_{t-1} + \underbrace{\mathbb{E}(u_t \mid Y_{t-1}, \dots)}_{=0}$$

$$\mathbb{E}(u_t \mid Y_{t-1}, Y_{t-2}, \dots) = 0$$

* $\mathbb{E}(u_t \mid Y_{t-1}, Y_{t-2}, \dots) > 0$
* $\mathbb{E}(u_t \mid Y_{t-1}, Y_{t-2}, \dots) < 0$