Exercise 1

final wealth after one trading day $\quad 1000(1+R_t)$

$$\mathbb{P}(1000(1+R_t) \le 990)$$

r_t's $\overset{\text{IID}}{\sim} N(0.001, 0.015^2)$

$\downarrow \log(1+R_t)$

$1000(1+R_t) \le 990$

$\Rightarrow \quad 1+R_t \le 0.99$

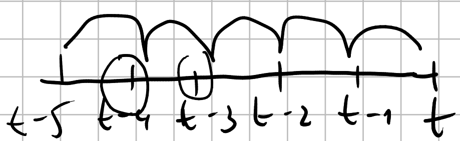
$\Rightarrow \quad \underbrace{\log(1+R_t)}_{r_t} \le \log(0.99)$

$$\mathbb{P}(1000(1+R_t) \le 990) = \mathbb{P}(r_t \le \log(0.99))$$

$\approx -0.73 \sim -0.74$

$$= \mathbb{P}\left(\frac{r_t - 0.001}{0.015} \le \frac{\log(0.99) - 0.001}{0.015}\right)$$

$\approx 0.23 \qquad$ pnorm $\rightarrow$ 0.2306557



(b) final wealth after 5 trading days

$= 1000(1+R_t(5))$

$$\mathbb{P}(1000(1+R_t(5)) \le 990) \qquad \text{log returns}$$

$1000(1+R_t(5)) \le 990 \Rightarrow \quad 1+R_t(5) \le 0.99$

$\Rightarrow \quad \log(1+R_t(5)) \le \log(0.99)$

$\Rightarrow \quad r_t + r_{t-1} + r_{t-2} + r_{t-3} + r_{t-4} \le \log(0.99)$

$$= \mathbb{P}(r_t + \dots + r_{t-4} \le \log(0.99))$$

$r_t + \dots + r_{t-4} \sim N(5 \cdot 0.001, 5 \cdot 0.015^2)$

$= 0.3268189$

Practice Exercises $\quad 2, 3, \underbrace{4, 5, 6}_{\text{no RW model}}, 7a, 7b.$

How would one answer Exercise 1 completely using a computer alone?

$\rightarrow$ Section 2.4.3

$$P_t = P_0 \exp(r_t + r_{t-1} + \dots + r_1)$$

$$P_0, \quad P_1 = P_0 \exp(r_1), \quad P_2 = P_0 \exp(r_1 + r_2)$$

r_t's $\overset{iid}{\sim} N(\mu, \sigma^2)$

$r_t + \cdot + r_{t-k+1} \sim N(k\mu, k\sigma^2)$

Section 2.4.3

$\mathbb{E}(r_t + \cdots + r_{t-k+1}) = k\mu = 253\mu = 0.05$

$\mu = 0.05/253$

$Var(r_t + \cdots + r_{t-k+1}) = k\sigma^2$

$\sqrt{k}\sigma = 0.2$

$\sigma = 0.2/\sqrt{253}$

Exercise 7

log returns

$r_1, r_2, \ldots \overset{iid}{\sim} N(0.06, 0.47)$

$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$

(c) $Cov(r_2(1), r_2(2))$

$r_2(1) = r_2$

$r_2(2) = r_2 + r_1$

$= Cov(r_2, r_2 + r_1)$

$= Cov(r_2, r_2) + Cov(r_2, r_1) = Var(r_2) + 0 = 0.47$

X, Y, Z r.v.

$Cov(X, Y+Z)$

$= Cov(X, Y) + Cov(X, Z)$

X, Y are indept r.v. $\Rightarrow Cov(X, Y) = 0$

(d) $r_t(3) \mid r_{t-2} = 0.6$

$r_t(3) = r_{t-2} + r_{t-1} + r_t$

$\sim N(\underline{\quad}, \underline{\quad})$

$\mathbb{E}(r_{t-2} + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= \mathbb{E}(0.6 + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= 0.6 + \mathbb{E}(r_{t-1} \mid r_{t-2} = 0.6) + \mathbb{E}(r_t \mid r_{t-2} = 0.6)$

indept

$= 0.6 + \mathbb{E}(r_{t-1}) + \mathbb{E}(r_t)$

$= 0.6 + 0.06 + 0.06 = \underline{\quad}$

$Var(r_{t-2} + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= Var(0.6 + r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= Var(r_{t-1} + r_t \mid r_{t-2} = 0.6)$

$= Var(r_{t-1} + r_t)$

indept

$= Var(r_{t-1}) + Var(r_t)$

$= 2(0.47)$