#8

$X_k = X_0 \exp(r_1 + \dots + r_k) \qquad r_1, \dots \sim \text{iid } N(\mu, \sigma^2)$

(a) $P(X_2 > \underbrace{1.3 X_0}_{\text{constant}})$

$= P\left(\frac{X_2}{X_0} > 1.3\right)$

$= P(\exp(r_1 + r_2) > 1.3)$

$= P(r_1 + r_2 > \log(1.3)) = \underline{?}$

(b) $X_1 = X_0 \exp(\underbrace{r_1}_{\text{r.v.}}) \qquad r_1 \sim N(\mu, \sigma^2)$

Use (A.4)

$$\boxed{f_Y(y)} = f_X(h(y))\,|h'(y)|$$

$X_1 \to Y$ r.v. transformed/new

$r_1 \to X$ r.v. original/old $\xrightarrow{\text{know}}$ f_X density of X $r_1 \sim N(\mu, \sigma^2)$

$f_Y(y)$ density of Y at some value y

$f_X(h(y))$ density of X at some value $h(y)$

$Y = \underbrace{g}_{\text{strictly increasing}}(X) \Rightarrow X = \underbrace{\boxed{h}}_{\text{inverse of } g}(Y)$

$X_1 = \underbrace{X_0}_{>0} \underbrace{\exp(r_1)}_{>0}$

$g(u) = X_0 \exp(u)$

$g(r_1) = X_0 \exp(r_1) = X_1$

$\frac{X_1}{X_0} = \exp(r_1)$

$\log\left(\frac{X_1}{X_0}\right) = r_1$

$\underbrace{\log X_1 - \log X_0}_{h(X_1)} = r_1$

x_1

$\frac{dr_1}{dX_1} = \frac{1}{X_1}$

$f_Y(y) = f_X(h(y))\,|h'(y)|$

$\underbrace{f_{X_1}(x_1)}_{\substack{\text{density of } X_1 \\ \text{at some value } x_1}} = \underbrace{f_{r_1}(\log x_1 - \log X_0)}_{f_{r_1}(\square)} \left|\frac{1}{x_1}\right|$

Since $\underline{r_1 \sim N(\mu, \sigma^2)}$, $f_{r_1}(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(s-\mu)^2}{2\sigma^2}\right)$

$$\Rightarrow f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\log x_1 - \log X_0 - \mu)^2}{2\sigma^2}\right) \cdot \frac{1}{x_1}$$

named distributions / densities

$$= \frac{1}{x_1\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(\log x_1 - (\log X_0+\mu)\right)^2}{2\sigma^2}\right)$$

From the wiki, $\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)$ density of lognormal$(\underline{\mu},\underline{\sigma^2})$

density of lognormal$(\underline{\log X_0+\mu}, \sigma^2)$

$\Rightarrow X_1$ has a lognormal density with parameters $(\log X_0+\mu, \sigma^2)$.

An easier approach would be to exploit properties of lognormal:

$\to \quad X_1 = X_0 \exp(r_1)$

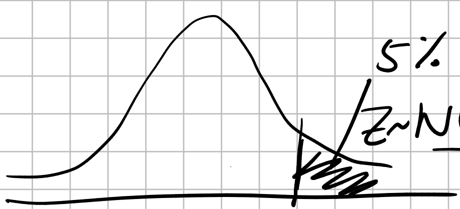
$\to \log\left(\frac{X_1}{X_0}\right) = r_1 \sim N(\mu,\sigma^2)$

$$\underbrace{\log X_1 = \underbrace{\boxed{\log X_0}}_{\text{constant}} + \underbrace{\boxed{r_1}}_{\sim N(\mu,\sigma^2)}}_{\sim N(\log \underline{X_0}+\mu,\ \sigma^2)}$$

$\Rightarrow$ A.9.4 $X_1 \sim$ Lognormal $(\log X_0+\mu, \sigma^2)$

#8 (c)

quantile



5%

$Z\sim N(0,1)$

$P(Z\le ?) = 0.95$

$? \leftarrow$ 0.95 quantile of $N(0,1)$

95th percentile

$X_k = X_0 \exp(\ldots)$

0.9 quantile of X_k $\qquad P(X_k \le x) = 0.9$

Chapter 4 Exploratory data analysis

Focus 4.1 - 4.5

How is this different from ECOMETR?

ECOMETR regressions.

- describe (summaries of a set of variables)
- prediction → conditional distributions.
- causality

statistical tools to highlight features of marginal distributions of r.v.'s

time series plots, histograms, density estimates, probability plots (QQ plots), boxplots

stationary

For every m, n $(Y_1, \ldots, Y_n)$ has the same dist as $(Y_{1+m}, \ldots, Y_{n+m})$

Set $m=1, n=1 \Rightarrow$ Y_1 has the same dist as Y_2

$m=2, n=1 \Rightarrow$ Y_1 has .. .. Y_3.

$m=1, n=2 \Rightarrow (Y_1, Y_2)$ .. .. as (Y_2, Y_3)