t_ν distribution has a zero popn mean by default, but the data on daily returns for PSEi has a nonzero sample mean

$X \sim t_\nu \xrightarrow{\text{transform}} Y = \mu + \lambda X$ (μ: location shift; λ: scaling factor)

linear transformation of r.v. X

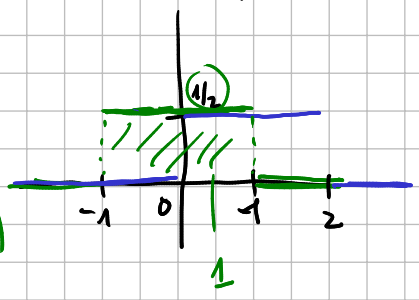
location-scale transformation (NOTE: change letters compared to book)

Example

$X \sim U(-1,1)$ → uniform dist

density of $U(-1,1)$

$E(X) = 0$

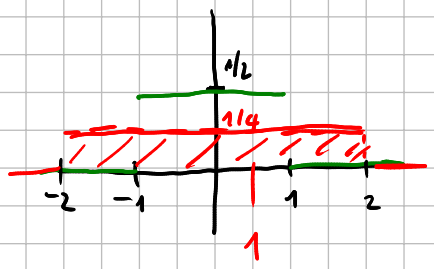


transform to Y

location change / shift

$E(Y) = 1$

Scale change



Result A.1 $Y = a + bX$ ($a = \mu$, $b = \lambda$)

$f_Y(y) = \frac{1}{|\lambda|} f_X\left(\frac{y-\mu}{\lambda}\right)$

density of t_ν

$Y \sim t_\nu(\mu, \lambda^2)$

ν: df; μ: popn mean if it exists; λ^2: scale

* Software implementations of t-dist can vary so pay attention to what was coded and available in software.

e.g. dnorm() → $N(\mu, \sigma)$ Books $N(\mu, \sigma^2)$

In R (fGarch package) dstd() density of standardized t-dist.

target: estimate parameters $t_\nu^{std}(\mu, \sigma^2)$ dist.

θ is one-dimensional 1×1 → θ multidimensional n×1

$I(\theta) = -E\left[\frac{d^2}{d\theta^2} \log L(\theta)\right]$

$I(\theta)$ Fisher info matrix

SE $\hat{\theta}$ ← sqrt(reciprocal of $I(\theta)$)

SE for each estimated qty ← sqrt(diag(inverse of Fisher information))

In R, (fGarch) dged() → density of generalized error dist

t-dist / GED → symmetric distributions by default

return dist may be asymmetric

introduce / incorporate asymmetry — Section 5.7

In R (fGarch), dsstd() skewed t-distribution

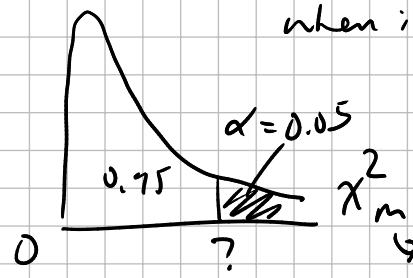
↑ skewed

$H_0: \xi = 1$ (symmetric)

$$\frac{\hat{\xi} - 1}{se(\hat{\xi})}$$

OR Likelihood ratio test

comparing log-likelihood without imposing the null against log-likelihood when imposing the null



↳ How many restrictions were imposed?