dependence + stationarity

↳ see Key Concept 15.3

Ruppert & Matteson

- strict stationarity ⇐ match — all about distributions.
- weak stationarity

strict stationarity + finite moments

$Y_1, Y_2, \ldots$ is a weakly stationary process

- $E(Y_t) = \mu$ regardless of the value t
- $Var(Y_t) = \sigma^2$ regardless of the value t
- $Cov(Y_t, Y_s) = \gamma(|t-s|)$

↳ γ is a function of $|t-s|$

gap between t & s.

strict stationarity + finite moments ⇒ weak stationarity

ECONSTA $\bar{Z} = \frac{1}{n}(Z_1 + Z_2 + \cdots + Z_n)$ $Z_1, \ldots, Z_n$ IID (μ, σ^2)

→ r.v. → dist

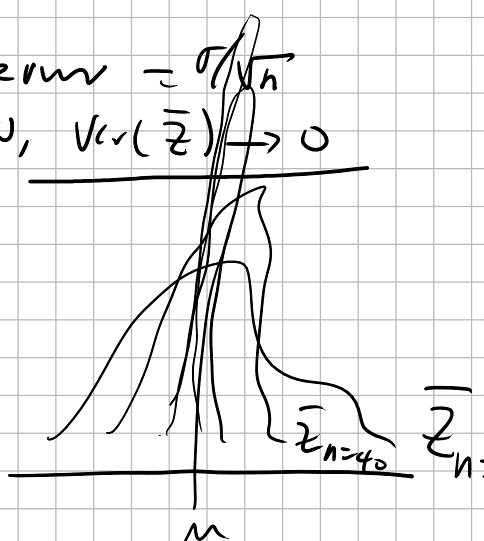
- $E(\bar{Z}) = \mu$
- $Var(\bar{Z}) = \sigma^2/n$ ⇒ standard error $= \sigma/\sqrt{n}$

As $n \to +\infty$, $Var(\bar{Z}) \to 0$

- $\bar{Z} \xrightarrow{p} \mu$

↳ converges in prob.

How will these results be affected by a change from IID to dependence + stationarity?



$$\bar{Z} = \frac{1}{n}(Z_1 + Z_2 + \cdots + Z_n) \quad n \text{ copies}$$

$$E(\bar{Z}) = \frac{1}{n}\left(E(Z_1) + E(Z_2) + \cdots + E(Z_n)\right)$$

stationarity $= \frac{1}{n}(n\mu) = \mu$

$E(Z_1) = E(Z_2) = \cdots = E(Z_n) = \mu = E(Z_t)$

Dependence is not an issue.

(version of idea seen in LL)

$$\mathrm{Var}(\bar{Z}) = \mathrm{Var}\left(\frac{1}{n}(Z_1+Z_2+\dots+Z_n)\right)$$

constant

$$= \frac{1}{n^2}\mathrm{Var}(Z_1+Z_2+\dots+Z_n)$$

stationarity → n copies

dependence is an issue here

$$= \frac{1}{n^2}\Big[\mathrm{Var}(Z_1)+\mathrm{Var}(Z_2)+\dots+\mathrm{Var}(Z_n)$$
$$+2\mathrm{Cov}(Z_1,Z_2)$$
$$+2\mathrm{Cov}(Z_1,Z_3)$$
$$\vdots \quad + \dots\Big]$$

$\mathrm{Var}(X+Y) = \mathrm{Var}(X)+\mathrm{Var}(Y)+2\mathrm{Cov}(X,Y)$

$\mathrm{Var}(X+Y+Z) = \mathrm{Var}(X)+\mathrm{Var}(Y)+\mathrm{Var}(Z)+2\mathrm{Cov}(X,Y)+2\mathrm{Cov}(X,Z)+2\mathrm{Cov}(Y,Z)$

$\mathrm{Var}(Z_1)=\mathrm{Var}(Z_2)=\mathrm{Var}(Z_3)=\dots=\mathrm{Var}(Z_n)=\sigma^2$

$$= \frac{1}{n^2}\left[n\sigma^2 + 2\mathrm{Cov}(Z_1,Z_2)+2\mathrm{Cov}(Z_1,Z_3)+\dots\right]$$

σ^2/n

$$\Rightarrow \mathrm{Var}(\bar{Z}) = \frac{\sigma^2}{n} + \dots$$

compare with IID case $\mathrm{Var}(\bar{Z}) = \sigma^2/n$

Organize calculations:

$$\begin{pmatrix}
\mathrm{Var}(Z_1) & \mathrm{Cov}(Z_1,Z_2) & \mathrm{Cov}(Z_1,Z_3) & \dots & \mathrm{Cov}(Z_1,Z_n)\\
\mathrm{Cov}(Z_2,Z_1) & \mathrm{Var}(Z_2) & \mathrm{Cov}(Z_2,Z_3) & \dots & \mathrm{Cov}(Z_2,Z_n)\\
\vdots & \mathrm{Cov}(Z_3,Z_2) & \vdots & & \vdots\\
\mathrm{Cov}(Z_n,Z_1) & \mathrm{Cov}(Z_n,Z_2) & \mathrm{Cov}(Z_n,Z_3) & \dots \ \mathrm{Cov}(Z_n,Z_{n-1}) & \mathrm{Var}(Z_n)
\end{pmatrix}$$

Because of stationarity $\mathrm{Cov}(Z_1,Z_2) = \mathrm{Cov}(Z_2,Z_3) = \dots$

$\mathrm{Cov}(Z_1,Z_3) = \mathrm{Cov}(Z_2,Z_4) = \dots$

$\mathrm{Cov}(Z_t,Z_{t-j}) = j$th order autocovariance

$1+2+3+4+\dots$ = no finite sum

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots = 2$

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots$ = no finite sum

- $\underline{Var(\bar{Z}) \to 0}$ we need autocovariances to be "controlled"
- What kind of time series models should we even consider?

$\underline{Y_t = 1 Y_{t-1} + u_t}$ random walk

$Y_t = \beta_1^* Y_{t-1} + u_t \quad 0 < \beta_1^* < 1$

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dependence + stationarity

points to the need to adjust standard errors relative to IID case

parallels to ECOMETRIC

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$$

$$E(\varepsilon_t | X_t) = 0$$

$$Var(\varepsilon_t \cancel{| X_t}) = \sigma^2 \text{ (cond. homoscedasticity)}$$

indept $\Rightarrow$ uncorrelated

$\not\Leftarrow$

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* SW: focus is on autoregressions, motivation is coming from the need to forecast, to apply regression methods in time series case

SDAPE: ~~focus~~ is in identifying what type of time series model is compatible with the data, forecasting a bit downplayed.

$1, 2, 3, \ldots, T$ $\quad$ $T+1$

what you observe $\quad$ you do not observe

Goal is to predict / forecast Y_{T+1} given

$\hat{Y}_{T+1|T} \rightarrow$ function of data from the past

min MSFE $\Rightarrow$ the best you could do is to

$$\underbrace{\mathbb{E}(Y_{T+1} | Y_T, \ldots, Y_1)}_{\text{conditional expectation}}$$

parallels to ECOMETR

	ECOFMET		ECOMETR	
what you see / observe	$Y_1, \ldots, Y_T$ the past	what you see/observe	$X_1, \ldots, X_k$	features / regressors / characteristics
what you want to predict / forecast	Y_{T+1}	what you want to predict	Y	
Best you could do	$\mathbb{E}(Y_{T+1} \| Y_T, \ldots, Y_1)$ Conditional Expectation	Best you could do	$\mathbb{E}(Y \| X_1, \ldots, X_k)$ Conditional exp.	

* population objects
inherently unknown $\rightarrow$ we have to estimate them
* general functions (we do not have specification for them)

parallels to ECONMETR

ECONMET (one possibility) | ECONMETR

Assume that (one possibility)

$$E(Y_{T+1} | Y_T, \dots, Y_1) = \beta_0 + \beta_1 Y_T$$

the best prediction of Y_{T+1} given its past is a linear function of the most recent value of Y

Assume that (one possibility)

$$E(Y | X_1, \dots, X_k) = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k$$

linear regression

$$\underbrace{E(Y_{T+1} | Y_T, \dots, Y_1)}_{\text{forecast}} = \beta_0 + \beta_1 Y_T$$

forecast error $= Y_{T+1} - E(Y_{T+1} | Y_T, \dots, Y_1)$

by assumption $= Y_{T+1} - (\beta_0 + \beta_1 Y_T)$

u_{T+1}

$$\Rightarrow Y_{T+1} = \beta_0 + \beta_1 Y_T + u_{T+1}$$

AR(1) $\quad Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$